Traditional Pathway: Geometry

TheThe fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean theorem in 8^{th} grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

Critical Area 6: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Units	Includes Standard Clusters*	Mathematical Practice Standards
Unit 1 Congruence, Proof, and Constructions	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Prove geometric theorems. • Make geometric constructions.	**Make sense of problems and persevere in solving them.** **Reason abstractly and quantitatively.** **Construct viable arguments and critique the reasoning of others.** **Model with mathematics.** **Use appropriate tools strategically.** **Attend to precision.** **Look for and make use of structure.** **Look for and express regularity in repeated reasoning.**
Unit 2 Similarity, Proof, and Trigonometry	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. • Define trigonometric ratios and solve problems involving right triangles. • Apply geometric concepts in modeling situations. • Apply trigonometry to general triangles.	
Unit 3 Extending to Three Dimensions	• Explain volume formulas and use them to solve problems. • Visualize the relation between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.	
Unit 4 Connecting Algebra and Geometry through Coordinates	• Use coordinates to prove simple geometric theorems algebraically. • Translate between the geometric description and the equation for a conic section.	
Unit 5 Circles With and Without Coordinates	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorem algebraically. • Apply geometric concepts in modeling situations.	
Unit 6 Applications of Probability	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model. • Use probability to evaluate outcomes of decisions.	

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Experiment with transformations in the plane. *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.*	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• Understand congruence in terms of rigid motions. *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.*	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
• Prove geometric theorems. *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 5.*	G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Make geometric constructions. *Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects.* *Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.*	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 2: Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Unit 2: Similarity, Proof, and Trigonometry	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand similarity in terms of similarity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
• Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*. G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
• Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
• Apply geometric concepts in modeling situations. *Focus on situations well modeled by trigonometric ratios for acute angles.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
• Apply trigonometry to general triangles. *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.*	G.SRT.9 (+) Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 3: Extending to Three Dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Unit 3: Extending to Three Dimensions	
Clusters and Instructional Notes	**Common Core State Standards**
• Explain volume formulas and use them to solve problems. *Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.*	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★
• Visualize the relation between two-dimensional and three-dimensional objects.	G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
• Apply geometric concepts in modeling situations. *Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Unit 4: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Use coordinates to prove simple geometric theorems algebraically. *This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* *Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in High School Algebra I involving systems of equations having no solution or infinitely many solutions.* *G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
• Translate between the geometric description and the equation for a conic section. *The directrix should be parallel to a coordinate axis.*	G.GPE.2 Derive the equation of a parabola given a focus and directrix.

Unit 5: Circles With and Without Coordinates

In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

Unit 5: Circles With and Without Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar. G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.
• Find arc lengths and areas of sectors of circles. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.*	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
• Translate between the geometric description and the equation for a conic section.	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
• Use coordinates to prove simple geometric theorems algebraically. *Include simple proofs involving circles.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
• Apply geometric concepts in modeling situations. *Focus on situations in which the analysis of circles is required.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

Unit 6: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Unit 6: Applications of Probability	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand independence and conditional probability and use them to interpret data. *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.*	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\vert A) = P(B)P(A\vert B)$, and interpret the answer in terms of the model. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
• Use probability to evaluate outcomes of decisions. *This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).